# ON THE STRUCTURE OF LEAD(II) COMPLEXES IN AQUEOUS SOLUTIONS. I. TRINUCLEAR CLUSTERS 

Martin Breza ${ }^{a}$ and Alena MANOVA ${ }^{b}$<br>${ }^{a}$ Department of Chemical Physics,<br>Slovak Technical University, 81237 Bratislava, The Slovak Republic<br>${ }^{b}$ Department of Analytical Chemistry,<br>Slovak Technical University, 81237 Bratislava, The Slovak Republic

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Using semiempirical MNDO method of quantum chemistry the optimal geometries and corresponding electronic structures of $\left[\mathrm{Pb}_{3}(\mathrm{OH})_{n}\right]^{6-n}$ model systems as well as of their hydrated $\left[\mathrm{Pb}_{3}(\mathrm{OH})_{n}\left(\mathrm{H}_{2} \mathrm{O}\right)_{8-n}\right]^{6-n}$ analogues $(n=4,5)$ are investigated. The most stable trinuclear lead(II) complexes present in aqueous solutions correspond to cyclo- $\left(\mu_{3}-\mathrm{OH}\right)\left(\mu_{2}-\mathrm{OH}\right)_{3} \mathrm{~Pb}_{3}^{2+}, \mathrm{Pb}(\mu-\mathrm{OH})_{2} \mathrm{~Pb}(\mu-\mathrm{OH})_{2} \mathrm{~Pb}^{2+}$, cyclo- $\left(\mu_{3}-\mathrm{OH}\right)_{2}\left(\mu_{2}-\mathrm{OH}\right)_{3} \mathrm{~Pb}_{3}^{+}, \mathrm{Pb}(\mathrm{OH})(\mu-\mathrm{OH})_{2} \mathrm{~Pb}(\mu-\mathrm{OH}) \mathrm{Pb}(\mathrm{OH})^{+}$and $\mathrm{Pb}(\mathrm{OH})(\mu-\mathrm{OH})_{2} \mathrm{~Pb}(\mu-\mathrm{OH})_{2} \mathrm{~Pb}^{+}$ systems. The key role of OH bridges (by vanishing direct $\mathrm{Pb}-\mathrm{Pb}$ bonds) on the stability of individual isomers is discussed.

Statistical analysis of potentiometric titration data ${ }^{1,2}$ on the hydrolysis of lead(II) indicates the formation of the species $[\mathrm{Pb}(\mathrm{OH})]^{+},\left[\mathrm{Pb}_{3}(\mathrm{OH})_{4}\right]^{2+},\left[\mathrm{Pb}_{3}(\mathrm{OH})_{5}\right]^{+},\left[\mathrm{Pb}_{4}(\mathrm{OH})_{4}\right]^{4+}$ and $\left[\mathrm{Pb}_{6}(\mathrm{OH})_{8}\right]^{4+}$.

The dominant hydrolytic species ${ }^{1,2}$ at low lead(II) concentrations is $[\mathrm{Pb}(\mathrm{OH})]^{+}$. Other monomeric species as $\left[\mathrm{Pb}(\mathrm{OH})_{2}\right]$ and $\left[\mathrm{Pb}(\mathrm{OH})_{3}\right]^{-}$are not produced at pH values lower than 8.5.

Among the dimeric species $\left[\mathrm{Pb}_{2}(\mathrm{OH})\right]^{3+}$ was not considered as making any measurable contribution to total lead concentration in aqueous solution ${ }^{1}$. The absence of the species $\left[\mathrm{Pb}_{2}(\mathrm{OH})_{2}\right]^{2+}$ is possibly the result of dimerization reaction being heavily forwarded to $\left[\mathrm{Pb}_{4}(\mathrm{OH})_{4}\right]^{4+}$ formation.

The species $\left[\mathrm{Pb}_{3}(\mathrm{OH})_{4}\right]^{2+}$ is present in aqueous solution to a considerable extent ${ }^{1}$. Its contribution to total lead concentration in solution is more important than this contribution of $\left[\mathrm{Pb}_{4}(\mathrm{OH})_{4}\right]^{4+}$ and $\left[\mathrm{Pb}_{6}(\mathrm{OH})_{8}\right]^{4+}$ species at any pH where all three exist. At high pH the concentration of $\left[\mathrm{Pb}_{3}(\mathrm{OH})_{5}\right]^{+}$species becomes one of the highest ones, even at low total lead concentrations.

There are no experimental data on the structure of the above mentioned trinuclear lead(II) clusters in aqueous solution (on the contrary to tetra- and hexanuclear ones) ${ }^{2}$. Their structure may be deduced from analogies with similar compounds but some questions remain unresolved. Consequently, the systematic study of the structure of above
mentioned lead(II) clusters by means of quantum chemistry is desirable. The aim of this study is to determine the structure of $\left[\mathrm{Pb}_{3}(\mathrm{OH})_{4}\right]^{2+}$ and $\left[\mathrm{Pb}_{3}(\mathrm{OH})_{5}\right]^{+}$clusters present in aqueous solution.

## EXPERIMENTAL

## Method Used

The standard semiempirical MNDO (Modified Neglect of Diatomic Overlap) method of quantum chemistry (AMPAC program package) ${ }^{3-5}$ has been used in order to find the optimal geometries and to calculate the corresponding electronic structures of $\left[\mathrm{Pb}_{3}(\mathrm{OH})_{4}\right]^{2+}$ (models $\mathbf{A}_{m n}$ and $\mathbf{B}_{p q}$ ) $\left[\mathrm{Pb}_{3}(\mathrm{OH})_{5}\right]^{+}\left(\right.$models $\mathbf{E}_{m n}$ and $\left.\mathbf{F}_{p q}\right)$ clusters as well as of their hydrated analogs $\left[\mathrm{Pb}_{3}(\mathrm{OH})_{4}\left(\mathrm{H}_{2} \mathrm{O}\right)_{4}\right]^{2+}$ (models $\mathbf{C}_{m n}$ and $\mathbf{D}_{p q}$ ) and $\left[\mathrm{Pb}_{3}(\mathrm{OH})_{5}\left(\mathrm{H}_{2} \mathrm{O}\right)_{3}\right]^{+}\left(\right.$models $\mathbf{G}_{m n}$ and $\mathbf{H}_{p q}$ ). In the cyclic $\mathbf{A}_{m n}, \mathbf{C}_{m n}, \mathbf{E}_{m n}$ and $\mathbf{G}_{m n}$ models (triangular $\mathrm{Pb}_{3}$ geometry - see Figs 1, 2) the $m$ and $n$ indices denote the numbers of $\mu_{3^{-}}$ and $\mu_{2}$-OH bridges, respectively. In the acyclic $\mathbf{B}_{p q}, \mathbf{D}_{p q}, \mathbf{F}_{p q}$ and $\mathbf{H}_{p q}$ (quasi-linear $\mathrm{Pb}_{3}$ geometry see Figs $3,4,5$ ) the $p$ and $q$ indices denote the numbers of $\mu_{2}-\mathrm{OH}$ bridges on different $\mathrm{Pb}-\mathrm{Pb}$ bonds, respectively. The results are compared in terms like heat of formation $\left(\Delta H_{\mathrm{f}}\right)$, geometry changes, ionisation energy ( $I$ ) as well as in terms of Mulliken population analysis - bond orders (BO) and atomic charges $(Q)$. All calculations were performed in higher precision (keyword PRECISE) using Davi-don-Fletcher-Powell optimization procedure ${ }^{6,7}$.

## RESULTS AND DISCUSSION

Due to the large number of the studied model systems, some results for the stable systems only are presented in Tables I - IV. Geometries of the most stable model systems are presented in Figs $1-5\left(\mathrm{H}_{2} \mathrm{O}\right.$ molecules are omitted) and selected characteristics of corresponding electronic structures are described in Tables V - VII. It is evident that individual Pb atoms must be connected by OH bridges in order to produce stable structures. In principle, the increasing stability of the system with the number of OH bridges may be explained by increasing number of stabilizing $\mathrm{Pb}-\mathrm{O}$ bonds. In cyclic systems, however, the stabilization by $\mu_{2}-\mathrm{OH}$ and $\mu_{3}-\mathrm{OH}$ bridges are in contra-


Fig. 1
Stable geometry of cyclo- $\left(\mu_{3}-\mathrm{OH}\right)\left(\mu_{2}-\mathrm{OH}\right)_{3} \mathrm{~Pb}_{3}^{2+}$ (model $\mathbf{A}_{13}$ )
diction (compare stable $\mathbf{A}_{20}$ and unstable $\mathbf{A}_{21}$ models in Table I or $\mathbf{E}_{20}$ and less stable $\mathbf{E}_{21}$ ones in Table III). Really, $\mu_{2}-\mathrm{OH}$ bridges cause the increasing $\mathrm{Pb}-\mathrm{Pb}$ interatomic distances whereas the $\mu_{3}-\mathrm{OH}$ ones cause, in general, the decrease of this distance (see the $\mathrm{Pb}-\mathrm{Pb}$ distances in Tables I-IV). As a consequence the model systems $\mathbf{A}_{13}$ and $\mathbf{C}_{13}$ are more stable than their $\mathbf{A}_{22}$ and $\mathbf{C}_{22}$ analogues. Additional OH bridge in acyclic model systems causes the shortening of $\mathrm{Pb}-\mathrm{Pb}$ distances (see Tables I - IV). The decrease of the total cluster charge (rising electron density) causes the shortening of $\mathrm{Pb}-$ Pb distances, too.

Table I
Heat of formation, $\Delta H_{\mathrm{f}}$, and mutual $\mathrm{Pb}-\mathrm{Pb}$ distances, $d(\mathrm{~Pb}-\mathrm{Pb})$, of stable $\mathrm{Pb}_{3}(\mathrm{OH})_{4}^{2+}$ systems

| Model | Cluster | $\Delta H_{\mathrm{f}}, \mathrm{kJ} / \mathrm{mol}$ | $d(\mathrm{~Pb}-\mathrm{Pb}), 10^{-10} \mathrm{~m}$ |
| :---: | :---: | :---: | :---: |
| Cyclic models |  |  |  |
| $\mathbf{A}_{02}$ | cyclo- $\left(\mu_{2}-\mathrm{OH}\right)_{2} \mathrm{~Pb}_{3}(\mathrm{OH})_{2}^{2+}$ | +1 077.7 | 3.87, 3.40, 3.92 |
| $\mathbf{A}_{03}$ | cyclo- $\left(\mu_{2}-\mathrm{OH}\right)_{3} \mathrm{~Pb}_{3}(\mathrm{OH})^{2+}$ | +858.6 | 4.26 (2×), 4.04 |
| $\mathbf{A}_{12}$ | cyclo- $\left(\mu_{3}-\mathrm{OH}\right)\left(\mu_{2}-\mathrm{OH}\right)_{2} \mathrm{~Pb}_{3}(\mathrm{OH})^{2+}$ | +896.6 | 3.70, 3.87, 3.55 |
| $\mathbf{A}_{13}$ | cyclo- $\left(\mu_{3}-\mathrm{OH}\right)\left(\mu_{2}-\mathrm{OH}\right)_{3} \mathrm{~Pb}_{3}^{2+}$ | +567.9 | 3.62 ( $3 \times$ ) |
| $\mathbf{A}_{20}$ | cyclo $-\left(\mu_{3}-\mathrm{OH}\right)_{2} \mathrm{~Pb}_{3}(\mathrm{OH})_{2}^{2+}$ | +1 042.5 | 3.52, 3.53, 3.95 |
| $\mathbf{A}_{22}$ | cyclo- $\left(\mu_{3}-\mathrm{OH}\right)_{2}\left(\mu_{2}-\mathrm{OH}\right)_{2} \mathrm{~Pb}_{3}^{2+}$ | +680.4 | 3.32, $3.49(2 \times)$ |
| Acyclic models |  |  |  |
| $\mathbf{B}_{11}$ | $\mathrm{Pb}(\mathrm{OH})(\mu-\mathrm{OH}) \mathrm{Pb}(\mu-\mathrm{OH}) \mathrm{Pb}(\mathrm{OH})^{2+}$ | +760.5 | $4.00(2 \times), 7.69$ |
| $\mathbf{B}_{21}$ | $\mathrm{Pb}\left((\mu-\mathrm{OH})_{2} \mathrm{~Pb}(\mu-\mathrm{OH}) \mathrm{Pb}(\mathrm{OH})^{2+}\right.$ | +639.8 | 3.55, 3.96, 6.79 |
| $\mathbf{B}_{22}$ | $\mathrm{Pb}(\mu-\mathrm{OH})_{2} \mathrm{~Pb}(\mu-\mathrm{OH})_{2} \mathrm{~Pb}^{2+}$ | +595.6 | $3.59(2 \times), 6.21$ |

Fig. 2
Stable geometry of $\mathrm{Pb}(\mu-\mathrm{OH})_{2} \mathrm{~Pb}(\mu-\mathrm{OH})_{2} \mathrm{~Pb}^{2+}$ (model $\mathbf{B}_{22}$ )


Inclusion of additional $\mathrm{H}_{2} \mathrm{O}$ molecules into the model may significantly influence the stability of the systems under study (compare relative stabilities of $\mathbf{A}, \mathbf{B}, \mathbf{E}$ and $\mathbf{F}$ models in Tables I and III with their hydrated $\mathbf{C}, \mathbf{D}, \mathbf{G}$ and $\mathbf{H}$ analogues in Tables II and IV). Here it must be mentioned that the influence of polar $\mathrm{H}_{2} \mathrm{O}$ molecules on the acyclic systems is greater than on cyclic ones (due to higher polarity). More complex models (accounting a greater number of $\mathrm{H}_{2} \mathrm{O}$ molecules) may cause larger stability differences between cyclic and acyclic systems. Due to this reason the most stable systems for both the model types are presented in Tables V - VII. Geometry investigations of $\mathbf{C}, \mathbf{D}, \mathbf{G}$ and $\mathbf{H}$ models (due to space shortage not presented in details) indicate that $\mathrm{H}_{2} \mathrm{O}$ ligands

Table II
Heat of formation, $\Delta H_{\mathrm{f}}$, and mutual $\mathrm{Pb}-\mathrm{Pb}$ distances, $d(\mathrm{~Pb}-\mathrm{Pb})$, of stable $\mathrm{Pb}_{3}(\mathrm{OH})_{4}\left(\mathrm{H}_{2} \mathrm{O}\right)_{4}^{2+}$ systems

| Model | Cluster | $\underset{\mathrm{kJ} / \mathrm{mol}}{\Delta H_{\mathrm{f}},}$ | $d(\mathrm{~Pb}-\mathrm{Pb}), 10^{-10} \mathrm{~m}$ |
| :---: | :---: | :---: | :---: |
| Cyclic models |  |  |  |
| $\mathrm{C}_{02}$ | cyclo- $\left(\mu_{3}-\mathrm{H}_{2} \mathrm{O}\right)_{2}\left(\mu_{2}-\mathrm{OH}\right)_{2} \mathrm{~Pb}_{3}(\mathrm{OH})_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{2}^{2+}$ | -424.9 | 4.10, 4.20, 3.99 |
| $\mathrm{C}_{12}$ | cyclo- $\left(\mu_{3}-\mathrm{OH}\right)\left(\mu_{3}-\mathrm{H}_{2} \mathrm{O}\right)\left(\mu_{2}-\mathrm{OH}\right)_{2}\left(\mu_{2}-\mathrm{H}_{2} \mathrm{O}\right) \mathrm{Pb}_{3}(\mathrm{OH})\left(\mathrm{H}_{2} \mathrm{O}\right) 2^{2+}$ | -271.1 | 3.47, 3.67, 3.45 |
| $\mathrm{C}_{13}$ | cyclo- $\left(\mu_{3}-\mathrm{OH}\right)\left(\mu_{3}-\mathrm{H}_{2} \mathrm{O}\right)\left(\mu_{2}-\mathrm{OH}\right)_{3} \mathrm{~Pb}_{3}\left(\mathrm{H}_{2} \mathrm{O}\right)_{3}^{2+}$ | -573.5 | $3.74(2 \times), 3.72$ |
| $\mathrm{C}_{20}$ | cyclo- $\left(\mu_{3}-\mathrm{OH}\right)_{2} \mathrm{~Pb}_{3}(\mathrm{OH})_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{4}^{2+}$ | -114.6 | 3.53, 3.54, 3.92 |
| $\mathrm{C}_{22}$ | cyclo- $\left(\mu_{3}-\mathrm{OH}\right)_{2}\left(\mu_{2}-\mathrm{OH}\right)_{2}\left(\mu_{2}-\mathrm{H}_{2} \mathrm{O}\right) \mathrm{Pb}_{3}(\mathrm{OH})\left(\mathrm{H}_{2} \mathrm{O}\right)_{3}^{2+}$ | -439.7 | 3.32, 3.43 ( $2 \times$ ) |
| Acyclic models |  |  |  |
| $\mathrm{D}_{11}$ | $\mathrm{Pb}(\mathrm{OH})\left(\mathrm{H}_{2} \mathrm{O}\right)(\mu-\mathrm{OH}) \mathrm{Pb}\left(\mathrm{H}_{2} \mathrm{O}\right)_{2}(\mu-\mathrm{OH}) \mathrm{Pb}(\mathrm{OH})\left(\mathrm{H}_{2} \mathrm{O}\right)^{2+}$ | -519.4 | $4.00(2 \times), 7.78$ |
| $\mathrm{D}_{21}$ | $\mathrm{Pb}\left(\mathrm{H}_{2} \mathrm{O}\right)_{2}(\mu-\mathrm{OH})_{2} \mathrm{~Pb}\left(\mathrm{H}_{2} \mathrm{O}\right)(\mu-\mathrm{OH}) \mathrm{Pb}(\mathrm{OH})\left(\mathrm{H}_{2} \mathrm{O}\right)^{2+}$ | -599.9 | 3.59, 3.92, 6.85 |
| $\mathrm{D}_{22}$ | $\mathrm{Pb}\left(\mathrm{H}_{2} \mathrm{O}\right)_{2}(\mu-\mathrm{OH})_{2} \mathrm{~Pb}(\mu-\mathrm{OH})_{2} \mathrm{~Pb}\left(\mathrm{H}_{2} \mathrm{O}\right)_{2}^{2+}$ | -635.5 | $3.62(2 \times), 6.28$ |



Fig. 3
Stable geometry of cyclo- $\left(\mu_{3}-\mathrm{OH}\right)_{2}\left(\mu_{2}-\mathrm{OH}\right)_{3} \mathrm{~Pb}_{3}^{+}$ (model $\mathbf{E}_{23}$ )
do not serve as bridges, their distance from Pb atoms is over $2.4 .10^{-10} \mathrm{~m}$ (compare with metalic Pb , covalent $\mathrm{Pb}(\mathrm{II})$ and ionic $\mathrm{Pb}^{2+}$ radii of $1.75 .10^{-10} \mathrm{~m}, 1.44 .10^{-10} \mathrm{~m}$ and $1.21 .10^{-10} \mathrm{~m}$, respectively $)^{2}$. The number of $\mathrm{H}_{2} \mathrm{O}$ ligands bonded to Pb atoms is restricted so that the number of $\mathrm{Pb}-\mathrm{O}$ bonds (including OH ligands) is limited to four. The remaining $\mathrm{H}_{2} \mathrm{O}$ molecules are bonded in a second coordination sphere.

Table III
Heat of formation, $\Delta H_{\mathrm{f}}$, and mutual $\mathrm{Pb}-\mathrm{Pb}$ distances, $d(\mathrm{~Pb}-\mathrm{Pb})$, of stable $\mathrm{Pb}_{3}(\mathrm{OH})_{5}^{+}$systems

| Model | Cluster | $\Delta H_{\mathrm{f}}, \mathrm{kJ} / \mathrm{mol}$ | $d(\mathrm{~Pb}-\mathrm{Pb}), 10^{-10} \mathrm{~m}$ |
| :---: | :---: | :---: | :---: |
| Cyclic models |  |  |  |
| $\mathbf{E}_{02}$ | cyclo- $\left(\mu_{2}-\mathrm{OH}\right)_{2} \mathrm{~Pb}_{3}(\mathrm{OH})_{3}^{+}$ | -204.0 | $2.72(2 \times), 3.40$ |
| $\mathbf{E}_{03}$ | cyclo- $\left(\mu_{2}-\mathrm{OH}\right)_{3} \mathrm{~Pb}_{3}(\mathrm{OH})_{2}^{+}$ | -193.9 | $4.02(2 \times), 4.12$ |
| $\mathbf{E}_{11}$ | cyclo- $\left(\mu_{3}-\mathrm{OH}\right)\left(\mu_{2}-\mathrm{OH}\right) \mathrm{Pb}_{3}(\mathrm{OH})^{+}$ | -125.2 | $3.26(2 \times), 3.29$ |
| $\mathbf{E}_{12}$ | cyclo- $\left(\mu_{3}-\mathrm{OH}\right)\left(\mu_{2}-\mathrm{OH}\right)_{2} \mathrm{~Pb}_{3}(\mathrm{OH})_{2}^{+}$ | -166.1 | 3.51, 3.17, 3.70 |
| $\mathbf{E}_{13}$ | $\text { cyclo- }\left(\mu_{3}-\mathrm{OH}\right)\left(\mu_{2}-\mathrm{OH}\right)_{3} \mathrm{~Pb}_{3}(\mathrm{OH})^{+}$ | -254.4 | $3.81(2 \times), 3.58$ |
| $\mathbf{E}_{20}$ | cyclo- $\left(\mu_{3}-\mathrm{OH}\right)_{2} \mathrm{~Pb}_{3}(\mathrm{OH})_{3}^{+}$ | -114.4 | 3.55 ( $3 \times$ ) |
| $\mathbf{E}_{21}$ | cyclo- $\left(\mu_{3}-\mathrm{OH}\right)_{2}\left(\mu_{2}-\mathrm{OH}\right) \mathrm{Pb}_{3}(\mathrm{OH})_{2}^{+}$ | -90.5 | $3.24(2 \times), 3.64$ |
| $\mathbf{E}_{22}$ | cyclo- $\left(\mu_{3}-\mathrm{OH}\right)_{2}\left(\mu_{2}-\mathrm{OH}\right)_{2} \mathrm{~Pb}_{3}(\mathrm{OH})^{+}$ | -309.4 | 3.66 ( $2 \times$ ), 3.10 |
| $\mathbf{E}_{23}$ | cyclo- $\left(\mu_{3}-\mathrm{OH}\right)_{2}\left(\mu_{2}-\mathrm{OH}\right)_{3} \mathrm{~Pb}_{3}^{+}$ | -493.8 | 3.48 ( $3 \times$ ) |
| Acyclic models |  |  |  |
| $\mathrm{F}_{11}$ | $\mathrm{Pb}(\mathrm{OH})(\mu-\mathrm{OH}) \mathrm{Pb}(\mathrm{OH})(\mu-\mathrm{OH}) \mathrm{Pb}(\mathrm{OH})^{+}$ | -428.7 | 3.90 ( $2 \times$ ), 6.81 |
| $\mathrm{F}_{21}$ | $\mathrm{Pb}(\mathrm{OH})(\mu-\mathrm{OH})_{2} \mathrm{~Pb}(\mu-\mathrm{OH}) \mathrm{Pb}(\mathrm{OH})^{+}$ | -521.1 | 3.54, 3.90, 6.61 |
| $\mathrm{F}_{22}$ | $\mathrm{Pb}(\mathrm{OH})(\mu-\mathrm{OH})_{2} \mathrm{~Pb}(\mu-\mathrm{OH})_{2} \mathrm{~Pb}^{+}$ | -508.7 | 3.59, 3.62, 5.75 |

Fig. 4
Stable geometry of $\mathrm{Pb}(\mathrm{OH})(\mu-\mathrm{OH})_{2} \mathrm{~Pb}(\mu-\mathrm{OH})$
$\mathrm{Pb}(\mathrm{OH})^{+}\left(\right.$model $\left.\mathbf{F}_{21}\right)$


The geometry of the most stable cyclic systems (due to space shortage not presented in details) corresponds to the highest possible symmetry ( $C_{3 \mathrm{v}}$ ) symmetry group for $\mathbf{A}_{13}$ and $\mathbf{C}_{13}$ models, $D_{3 \mathrm{~h}}$ for $\mathbf{E}_{23}$ and $\mathbf{G}_{23}$ ones). On the contrary, this rule does not hold for the acyclic systems where the $\mathrm{Pb}-\mathrm{Pb}-\mathrm{Pb}$ angle is near $120^{\circ}$ (non-linear systems). Nonbridging $\mathrm{Pb}-\mathrm{O}$ bonds (under $2.1 .10^{-10} \mathrm{~m}$ ) are shorter than the bridging ones, the

Table IV
Heat of formation, $\Delta H_{\mathrm{f}}$, and mutual $\mathrm{Pb}-\mathrm{Pb}$ distances, $d(\mathrm{~Pb}-\mathrm{Pb})$, of stable $\mathrm{Pb}_{3}(\mathrm{OH})_{5}\left(\mathrm{H}_{2} \mathrm{O}\right)_{3}^{+}$systems

Model Cluster | $\Delta H_{\mathrm{f}}$, |
| :---: |
| $\mathrm{kJ} / \mathrm{mol}$ |$d(\mathrm{~Pb}-\mathrm{Pb}), 10^{-10} \mathrm{~m}$

Cyclic model

| $\mathbf{G}_{02}$ | cyclo $-\left(\mu_{3}-\mathrm{H}_{2} \mathrm{O}\right)_{2}\left(\mu_{2}-\mathrm{OH}\right)_{2}\left(\mu_{2}-\mathrm{H}_{2} \mathrm{O}\right) \mathrm{Pb}_{3}(\mathrm{OH})_{3}^{+}$ | -1007.1 | $3.70(2 \times), 3.39$ |
| :--- | :--- | ---: | :--- |
| $\mathbf{G}_{03}$ | cyclo $-\left(\mu_{3}-\mathrm{H}_{2} \mathrm{O}\right)_{2}\left(\mu_{2}-\mathrm{OH}\right)_{3} \mathrm{~Pb}_{3}(\mathrm{OH})_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)^{+}$ | -990.0 | $3.94(2 \times), 4.02$ |
| $\mathbf{G}_{11}$ | cyclo $-\left(\mu_{3}-\mathrm{OH}\right)\left(\mu_{3}-\mathrm{H}_{2} \mathrm{O}\right)\left(\mu_{2}-\mathrm{OH}\right)\left(\mu_{2}-\mathrm{H}_{2} \mathrm{O}\right)_{2} \mathrm{~Pb}_{3}(\mathrm{OH})_{3}^{+}$ | -930.4 | $3.27(3 \times)$ |
| $\mathbf{G}_{12}$ | cyclo $-\left(\mu_{3}-\mathrm{OH}\right)\left(\mu_{3}-\mathrm{H}_{2} \mathrm{O}\right)\left(\mu_{2}-\mathrm{OH}\right)_{2}\left(\mu_{2}-\mathrm{H}_{2} \mathrm{O}\right) \mathrm{Pb}_{3}(\mathrm{OH})_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)^{+}$ | -988.1 | $3.44,3.13,3.61$ |
| $\mathbf{G}_{13}$ | cyclo $-\left(\mu_{3}-\mathrm{OH}\right)\left(\mu_{3}-\mathrm{H}_{2} \mathrm{O}\right)\left(\mu_{2}-\mathrm{OH}\right)_{3} \mathrm{~Pb}_{3}(\mathrm{OH})\left(\mathrm{H}_{2} \mathrm{O}\right)_{2}^{+}$ | -1104.8 | $3.78,3.80,3.64$ |
| $\mathbf{G}_{20}$ | cyclo $-\left(\mu_{3}-\mathrm{OH}\right)_{2} \mathrm{~Pb}_{3}(\mathrm{OH})_{3}\left(\mathrm{H}_{2} \mathrm{O}\right)_{3}^{+}$ | -911.5 | $3.54(3 \times)$ |
| $\mathbf{G}_{21}$ | cyclo $-\left(\mu_{3}-\mathrm{OH}\right)_{2}\left(\mu_{2}-\mathrm{OH}\right)\left(\mu_{2}-\mathrm{H}_{2} \mathrm{O}\right)_{2} \mathrm{~Pb}_{3}(\mathrm{OH})_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{2}^{+}$ | -896.9 | $3.23(2 \times), 3.60$ |
| $\mathbf{G}_{22}$ | cyclo $-\left(\mu_{3}-\mathrm{OH}\right)_{2}\left(\mu_{2}-\mathrm{OH}\right)_{2}\left(\mu_{2}-\mathrm{H}_{2} \mathrm{O}\right) \mathrm{Pb}_{3}(\mathrm{OH})\left(\mathrm{H}_{2} \mathrm{O}\right)_{2}^{+}$ | -1107.0 | $3.64(2 \times), 3.11$ |
| $\mathbf{G}_{23}$ | cyclo $-\left(\mu_{3}-\mathrm{OH}\right)_{2}\left(\mu_{2}-\mathrm{OH}\right)_{3} \mathrm{~Pb}_{3}\left(\mathrm{H}_{2} \mathrm{O}\right)_{3}^{+}$ | -1226.3 | $3.51(3 \times)$ |

Acyclic model
$\mathbf{H}_{21} \quad \mathrm{~Pb}(\mathrm{OH})\left(\mathrm{H}_{2} \mathrm{O}\right)(\mu-\mathrm{OH})_{2} \mathrm{~Pb}\left(\mathrm{H}_{2} \mathrm{O}\right)(\mu-\mathrm{OH}) \mathrm{Pb}(\mathrm{OH})\left(\mathrm{H}_{2} \mathrm{O}\right)^{+} \quad-1369.8 \quad 3.58,3.90,6.64$
$\mathbf{H}_{22} \mathrm{~Pb}(\mathrm{OH})\left(\mathrm{H}_{2} \mathrm{O}\right)(\mu-\mathrm{OH})_{2} \mathrm{~Pb}(\mu-\mathrm{OH})_{2} \mathrm{~Pb}\left(\mathrm{H}_{2} \mathrm{O}\right)_{2}^{+} \quad-1348.4 \quad$ 3.60, 3.62, 5.92


Fig. 5
Stable geometry of $\mathrm{Pb}(\mathrm{OH})(\mu-\mathrm{OH})_{2} \mathrm{~Pb}(\mu-$ $\mathrm{OH}){ }_{2} \mathrm{~Pb}^{+}\left(\right.$model $\left.\mathbf{F}_{22}\right)$

Table V
Electronic structure of the most stable $\mathrm{Pb}_{3}(\mathrm{OH})_{4}^{2+}$ and $\mathrm{Pb}_{3}(\mathrm{OH})_{4}\left(\mathrm{H}_{2} \mathrm{O}\right)_{4}^{2+}$ systems

| Model | $\mathbf{A}_{13}$ | $\mathbf{C}_{13}$ | $\mathbf{B}_{22}$ | $\mathbf{D}_{22}$ |
| :---: | :---: | :---: | :---: | :---: |
| I, eV | 20.0 | 16.9 | 19.0 | 17.1 |
| Atom charges |  |  |  |  |
| Pb | 1.07 (3×) | $\begin{aligned} & 0.97(2 \times) \\ & 0.94 \end{aligned}$ | $\begin{aligned} & 0.95^{a} \\ & 1.16(2 \times) \end{aligned}$ | $\begin{aligned} & 0.87^{a} \\ & 0.95(2 \times) \end{aligned}$ |
| O | $\begin{aligned} & -0.56(3 \times)^{b} \\ & -0.53 \end{aligned}$ | $\begin{aligned} & -0.56(3 \times)^{b} \\ & -0.52 \end{aligned}$ | $-0.55(4 \times)$ | -0.56(4×) |
| H | $\begin{aligned} & 0.25(3 \times)^{b} \\ & 0.24 \end{aligned}$ | $\begin{aligned} & 0.23(3 \times)^{b} \\ & 0.19 \end{aligned}$ | $0.24(4 \times)$ | 0.21 (4×) |
| Bond orders |  |  |  |  |
| $\mathrm{Pb}-\mathrm{Pb}$ | $0.02(3 \times)$ | 0.01 ( $3 \times$ ) | $0.02(2 \times$ ) | $0.02(2 \times$ ) |
| $\mathrm{Pb}-\mathrm{O}$ | $\begin{aligned} & 0.56(3 \times)^{b} \\ & 0.40(3 \times) \end{aligned}$ | $\begin{aligned} & 0.52(3 \times)^{b} \\ & 0.36(3 \times) \end{aligned}$ | $\begin{aligned} & 0.42(4 \times)^{a} \\ & 0.68(4 \times) \end{aligned}$ | $\begin{aligned} & 0.45(4 \times)^{a} \\ & 0.58(4 \times) \end{aligned}$ |
| O-H | $\begin{aligned} & 0.91(3 \times)^{b} \\ & 0.93 \end{aligned}$ | $\begin{aligned} & 0.93(3 \times)^{b} \\ & 0.95 \end{aligned}$ | 0.92 (4×) | 0.93 (4×) |

${ }^{a}$ Related to central $\mathrm{Pb} ;{ }^{b}$ related to $\mu_{2}-\mathrm{OH}$.

Model
Scheme

$$
\mathbf{F}_{21}, \mathbf{H}_{21}
$$


$\mathbf{F}_{22}, \mathbf{H}_{22}$


Fig. 6
Atom numbering for central part of acyclic models of stable $\mathrm{Pb}_{3}(\mathrm{OH})_{5}^{+}$and $\mathrm{Pb}_{3}(\mathrm{OH})_{5}\left(\mathrm{H}_{2} \mathrm{O}\right)_{3}^{+}$systems
$\mu_{2}$-ones (ca $2.2 .10^{-10} \mathrm{~m}$ ) shorter than the $\mu_{3}$-ones (ca $2.4 .10^{-10} \mathrm{~m}$ ). The influence of $\mathrm{H}_{2} \mathrm{O}$ molecules on the symmetry of optimal geometries is of lower importance.

On the other hand, the electronic structure of these optimal model geometries (Tables V - VII) is more influenced by $\mathrm{H}_{2} \mathrm{O}$ molecules. Ionisation potentials and polarity of atoms (especially Pb charges) are higher in simple $\mathbf{A}, \mathbf{B}, \mathbf{E}$ and $\mathbf{F}$ models than in their $\mathbf{C}, \mathbf{D}, \mathbf{G}$ and $\mathbf{H}$ analogues including additional $\mathrm{H}_{2} \mathrm{O}$ ligands. Inclusion of $\mathrm{H}_{2} \mathrm{O}$ influence especially the $\mathrm{Pb}-\mathrm{O}$ bonds and diminish the bond orders in cyclic models as well as in the $\mathrm{Pb}-\mathrm{O}$ bonds related to central Pb atom of acyclic models. Here it must be mentioned that the $\mathrm{Pb}-\mathrm{O}\left(\mathrm{H}_{2} \mathrm{O}\right)$ bond is relatively weak (bond orders ca $\left.0.2-0.3\right), \mu_{3}-\mathrm{OH}$ bridges in cyclic systems are bonded a little bit stronger (bond orders ca 0.4 ). The strongest $\mathrm{Pb}-\mathrm{O}$ bonds correspond to non-bridging OH in acyclic models (bond orders over 0.8). These data well correlate with corresponding bond lengths (shorter bond length corresponds to stronger bond and vice versa).

Due to vanishing $\mathrm{Pb}-\mathrm{Pb}$ bond order values the individual Pb atoms are bonded only via OH bridges. The increase of ionisation potential values with the positive charge of the system may be explained by increasing polarity of the system. By the same way is explainable the difference between cyclic and acyclic models. The decrease of ionisation potential values in hydrated models is, moreover, partly connected with the in-

Table VI
Electronic structure of the most stable cyclic $\mathrm{Pb}_{3}(\mathrm{OH})_{5}^{+}$and $\mathrm{Pb}_{3}(\mathrm{OH})_{5}\left(\mathrm{H}_{2} \mathrm{O}\right)_{3}^{+}$systems

| Model | $\mathbf{E}_{23}$ | $\mathbf{G}_{23}$ |
| :--- | :--- | :--- |
| $I, \mathrm{eV}$ | 14.5 | 13.1 |
|  | Atom charges |  |
| Pb | $0.90(3 \times)$ | $0.85(3 \times)$ |
| O | $-0.59(3 \times)^{a}$ | $-0.59(3 \times)^{a}$ |
|  | $-0.50(2 \times)$ | $-0.50(2 \times)$ |
| H | $+0.22(3 \times)^{a}$ | $+0.20(3 \times)^{a}$ |
|  | $+0.21(2 \times)$ | $+0.18(2 \times)$ |
|  | Bond orders |  |
| $\mathrm{Pb}-\mathrm{Pb}$ | $0.01(3 \times)$ | $0.01(3 \times)$ |
| $\mathrm{Pb}-\mathrm{O}$ | $0.49(3 \times)^{a}$ | $0.47(3 \times)^{a}$ |
|  | $0.38(2 \times)$ | $0.36(2 \times)$ |
| $\mathrm{O}-\mathrm{H}$ | $0.93(3 \times)^{a}$ | $0.94(3 \times)^{a}$ |
|  | $0.94(2 \times)$ | $0.95(2 \times)$ |

[^0]Table VII
Electronic structure of the most stable acyclic $\mathrm{Pb}_{3}(\mathrm{OH})_{5}^{+}$and $\mathrm{Pb}_{3}(\mathrm{OH})_{5}\left(\mathrm{H}_{2} \mathrm{O}\right)_{3}^{+}$systems (see Fig. 6 for atoms numbering)

| Model | $\mathbf{F}_{21}$ | $\mathbf{F}_{22}$ | $\mathbf{H}_{21}$ | $\mathbf{H}_{22}$ |
| :--- | :--- | :--- | :--- | :--- |
| $I, \mathrm{eV}$ | 13.5 | 13.0 | 13.2 | 12.6 |


| $\mathrm{Pb}(1)$ | 0.97 | 0.77 | 0.84 | 0.73 |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{~Pb}(2)$ | 0.89 | 0.86 | 0.88 | 0.83 |
| $\mathrm{~Pb}(3)$ | 0.83 | 1.05 | 0.81 | 0.92 |
| $\mathrm{O}(1)$ | $-0.53(2 \times)$ | $-0.53(2 \times)$ | $-0.52(2 \times)$ | $-0.53(2 \times)$ |
| $\mathrm{O}(2)$ | -0.52 | $-0.53(2 \times)$ | -0.52 | $-0.55(2 \times)$ |
| $\mathrm{O}(3)$ | -0.58 | -0.60 | -0.60 | -0.59 |
| $\mathrm{O}(4)$ | -0.61 | - | -0.61 | - |
| $\mathrm{H}(1)$ | $0.22(2 \times)$ | $0.23(2 \times)$ | $0.22(2 \times)$ | $0.20(2 \times)$ |
| $\mathrm{H}(2)$ | 0.24 | 0.18 | 0.23 | $0.22(2 \times)$ |
| $\mathrm{H}(3)$ | 0.22 | - | 0.20 | 0.17 |
| $\mathrm{H}(4)$ | 0.18 |  | 0.19 | - |

Bond orders

| $\mathrm{Pb}(1)-\mathrm{Pb}(2)$ | 0.02 | 0.02 | 0.02 | 0.02 |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{~Pb}(2)-\mathrm{Pb}(3)$ | 0.03 | 0.02 | 0.03 | 0.02 |
| $\mathrm{~Pb}(1)-\mathrm{O}(1)$ | $0.62(2 \times)$ | $0.51(2 \times)$ | $0.56(2 \times)$ | $0.55(2 \times)$ |
| $\mathrm{Pb}(2)-\mathrm{O}(1)$ | $0.61(2 \times)$ | $0.35(2 \times)$ | $0.57(2 \times)$ | $0.42(2 \times)$ |
| $\mathrm{Pb}(2)-\mathrm{O}(2)$ | 0.52 | $0.54(2 \times)$ | 0.61 | $0.50(2 \times)$ |
| $\mathrm{Pb}(3)-\mathrm{O}(2)$ | 0.48 | $0.76(2 \times)$ | 0.49 | $0.66(2 \times)$ |
| $\mathrm{Pb}(1)-\mathrm{O}(3)$ | 1.02 | 0.87 | 0.89 | 0.86 |
| $\mathrm{~Pb}(3)-\mathrm{O}(4)$ | 0.86 | - | 0.90 | - |
| $\mathrm{O}(1)-\mathrm{H}(1)$ | $0.93(2 \times)$ | $0.94(2 \times)$ | $0.93(2 \times)$ | $0.93(2 \times)$ |
| $\mathrm{O}(2)-\mathrm{H}(2)$ | 0.92 | $0.92(2 \times)$ | 0.92 | $0.94(2 \times)$ |
| $\mathrm{O}(3)-\mathrm{H}(3)$ | 0.92 | 0.95 | 0.94 | - |
| $\mathrm{O}(4)-\mathrm{H}(4)$ | 0.86 | - | 0.95 |  |

creasing size of the model systems. The above mentioned differences may indicate the possibility of verification of our results by electrochemical experiments.

Finally, it may be concluded that the most probable trilead(II) complexes in aqueous solutions correspond to cyclo- $\left(\mu_{3}-\mathrm{OH}\right)\left(\mu_{2}-\mathrm{OH}\right)_{3} \mathrm{~Pb}_{3}^{2+}, \mathrm{Pb}(\mu-\mathrm{OH})_{2} \mathrm{~Pb}(\mu-\mathrm{OH})_{2} \mathrm{~Pb}^{2+}$, cyclo-$\left(\mu_{3}-\mathrm{OH}\right)_{2}\left(\mu_{2}-\mathrm{OH}\right)_{3} \mathrm{~Pb}_{3}^{+}, \mathrm{Pb}(\mathrm{OH})(\mu-\mathrm{OH})_{2} \mathrm{~Pb}(\mu-\mathrm{OH}) \mathrm{Pb}(\mathrm{OH})^{+}$, and $\mathrm{Pb}(\mathrm{OH})(\mu-\mathrm{OH})_{2} \mathrm{~Pb}(\mu-\mathrm{OH})_{2} \mathrm{~Pb}^{+}$ systems. The key role of configuration of OH bridges on the stability of individual isomers is explained.

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[^0]:    ${ }^{a}$ Related to $\mu_{2}-\mathrm{OH}$.

